Algebra 2 Chapter 6 Review

Name: Key Hr: ______

For numbers 1 – 8 use the polynomial $f(x) = 2x^3 - 7x^2 - 5x + 6$

1. Name the function using the degree and number of terms:

   Cubic polynomial of 4 terms

2. What is the maximum number of x-intercepts?

   $\boxed{3}$

3. What is the end behavior of the graph?

   $\boxed{\downarrow \uparrow}$

5. The possible rational roots of the polynomial function are:

   $$\frac{\pm 1 \pm 2 \pm 3 \pm 6}{\pm 1 \pm 2} = \boxed{\pm 1, \pm 2, \pm 3, \pm 6, \pm \tfrac{1}{2}, \pm \tfrac{3}{2}}$$

6. How many zeros does the polynomial function have? How many could be imaginary? How many real?

   3 zeros
   0 or 2 could be imaginary
   1 or 3 could be real

7. Is $(x+1)$ a factor of the function?

   $$\begin{array}{r|rrrr} -1 & 2 & -7 & -5 & 6 \\ & & -2 & 9 & -4 \\ \hline & 2 & -9 & 4 & \boxed{2} \end{array}$$

   $\boxed{\text{no}}$

8. What is the value of the polynomial at f(8)? (use the reminder theorem, not a calculator!)

   $$\begin{array}{r|rrrr} 8 & 2 & -7 & -5 & 6 \\ & & 16 & 72 & 536 \\ \hline & 2 & 9 & 67 & \boxed{542} \end{array}$$

   $\boxed{f(8) = 542}$

Write a function $f(x)$ in standard form with the given zeros. Don't forget conjugates if needed.

9. -3, $7 - 4i$, $7+4i$

   $(x+3)(x-7+4i)(x-7-4i)$

   $(x+3)(x^2-7x-4ix-7x+49+28i+4ix-28i-16i^2)$

   $(x+3)(x^2-14x+65)$

   $x^3-14x^2+65x+3x^2-42x+195$

   $\boxed{x^3-11x^2+23x+195}$

10. 0 with multiplicity of 2, 5 and -1.

   $x^2(x-5)(x+1)$

   $x^2(x^2+x-5x-5)$

   $x^2(x^2-4x-5)$

   $\boxed{x^4-4x^3-5x^2}$

11. **SOLVING:** Find **all** the roots using any method. (Methods include perfect cube patterns, rational root theorem, graphing, grouping, factoring the old fashion way, etc…).

a) $3x^3+2x^2-37x+12=0$ $\quad \dfrac{\pm1 \pm2 \pm3 \pm4 \pm6 \pm12}{\pm1 \pm3}$

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

$$\begin{array}{r|rrrr} 3 & 3 & 2 & -37 & 12 \\ & & 9 & 33 & -12 \\ \hline & 3 & 11 & -4 & 0 \end{array}$$

$\boxed{x=3}$

$3x^2+11x-4=0$

$(3x-1)(x+4)=0$

$3x-1=0$ $\quad x+4=0$

$+1 \ +1$ $\quad -4 \ -4$

$\frac{3x}{3}=\frac{1}{3}$ $\quad \boxed{x=-4}$

$\boxed{x=\frac{1}{3}}$

b) $(20x^3-24x^2)(+25x-30)=0$

$4x^2(5x-6)+5(5x-6)=0$

$(4x^2+5)(5x-6)=0$

$4x^2+5=0$ $\quad 5x-6=0$

$-5 \ -5$ $\quad +6 \ +6$

$\frac{4x^2}{4}=\frac{-5}{4}$ $\quad \frac{5x}{5}=\frac{6}{5}$

$\sqrt{x^2}=\sqrt{\frac{-5}{4}}$ $\quad \boxed{x=\frac{6}{5}}$

$\boxed{x=\frac{\pm i\sqrt{5}}{2}}$

c) $x^4-19x^2-216=0$

$(x^2+8)(x^2-27)=0$

$x^2+8=0$ $\quad x^2-27=0$

$-8 \ -8$ $\quad +27 \ +27$

$\sqrt{x^2}=\sqrt{-8}$ $\quad \sqrt{x^2}=\sqrt{27}$

$\boxed{x=\pm 2i\sqrt{2}}$ $\quad \boxed{x=\pm 3\sqrt{3}}$

d) $x^6-25x^4=0$

$x^4(x^2-25)=0$

$x^4=0$ $\quad x^2-25=0$

$x\cdot x\cdot x\cdot x=0$ $\quad +25 \ +25$

$\boxed{x=0}$ $\quad \sqrt{x^2}=\sqrt{25}$

$\boxed{x=\pm5}$

e) $x^3-125=0$

$(x-5)(x^2+5x+25)=0$

$x-5=0$ $\quad x=\frac{-5\pm\sqrt{(5)^2-4(1)(25)}}{2(1)}$

$+5 \ +5$ $\quad =\frac{-5\pm\sqrt{25-100}}{2}$

$\boxed{x=5}$ $\quad =\frac{-5\pm\sqrt{-75}}{2}$

$=\boxed{\frac{-5\pm5i\sqrt{3}}{2}}$

f) $x^4-10x^2+24=0$

$(x^2-6)(x^2-4)=0$

$x^2-6=0$ $\quad x^2-4=0$

$+6 \ +6$ $\quad +4 \ +4$

$\sqrt{x^2}=\sqrt{6}$ $\quad \sqrt{x^2}=\sqrt{4}$

$\boxed{x=\pm\sqrt{6}}$ $\quad \boxed{x=\pm2}$

12. Multiply 3 - $\sqrt{6}$ by its conjugate and simplify.

$(3-\sqrt{6})(3+\sqrt{6})$

$9+3\sqrt{6}-3\sqrt{6}-\sqrt{36}$

$9-6$

$\boxed{3}$

13. Write an equation in standard form that has the solutions $3 \pm \sqrt{6}$. $3+\sqrt{6}$, $3-\sqrt{6}$

$(x-3-\sqrt{6})(x-3+\sqrt{6})=0$

$x^2-3x+\sqrt{6}x-3x+9-3\sqrt{6}-\sqrt{6}x+3\sqrt{6}-\sqrt{36}=0$

$\boxed{x^2-6x+3=0}$

14. Write an expression to represent the base of the rectangle shown below. Hint: How do you find area of a rectangle…now work backwards!

Height: $x-2$

Area = x^3-5x^2+8x-4

Base = $\boxed{x^2-3x+2}$

$$
\begin{array}{r|rrrr}
2 & 1 & -5 & 8 & -4 \\
 & & 2 & -6 & 4 \\
\hline
 & 1 & -3 & 2 & \boxed{0}
\end{array}
$$

x^2-3x+2

15. A polynomial equation has roots -2, 2+i and 1-3i. What is the least possible degree of the polynomial?

2-i 1+3i

$\boxed{5^{th} \text{ degree}}$

16. Using the equation answer true or false and explain why: $f(x)=10x^3-2x^2+5x-12$

a. 4 is a **possible root** of the equation.

True, because 4 is a factor of 12 and 1 is a factor of 10 so by using the Rational Root Theorem you get $\frac{4}{1}$ or 4 as a possible root.

b. 4 is one of the roots of the equation.

$$
\begin{array}{r|rrrr}
4 & 10 & -2 & 5 & -12 \\
 & & 40 & 152 & 628 \\
\hline
 & 10 & 38 & 157 & \boxed{616}
\end{array}
$$

False, when using synthetic division to divide 4 into the polynomial you get a remainder other than zero which means 4 is not a root

17. Divide $(4x^3+10x-5)\div(x-3)$ using synthetic division.

$$
\begin{array}{r|rrrr}
3 & 4 & 0 & 10 & -5 \\
 & & 12 & 36 & 138 \\
\hline
 & 4 & 12 & 46 & \boxed{133}
\end{array}
$$

$\boxed{4x^2+12x+46, R\ 133}$

18. Divide $3x^5 - 4x^3 + x - 10$ by $x^2 - 3$ using long division.

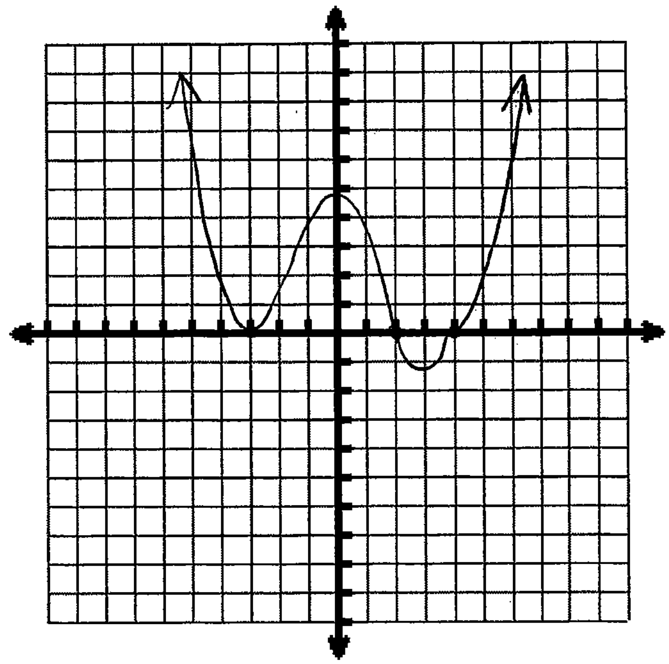
$$
\begin{array}{r}
3x^3 + 5x \\
x^2 - 3 \overline{)\,3x^5 - 4x^3 + x - 10} \\
\underline{-3x^5 + 9x^3} \\
5x^3 + x \\
\underline{-5x^3 + 15x} \\
16x - 10
\end{array}
$$

$\boxed{3x^3 + 5x, \ R\ 16x - 10}$

19. Graph a 6th degree polynomial function with a positive leading coefficient given its zeros are 2 with a multiplicity of one, 4 with a multiplicity of three, and -3 with a multiplicity of two.

$(x-2)(x-4)^3(x+3)^2$

End Behavior: ↑ ↑



Graph the polynomial. Find the x-intercepts, y-intercept, end behaviors and multiplicities.

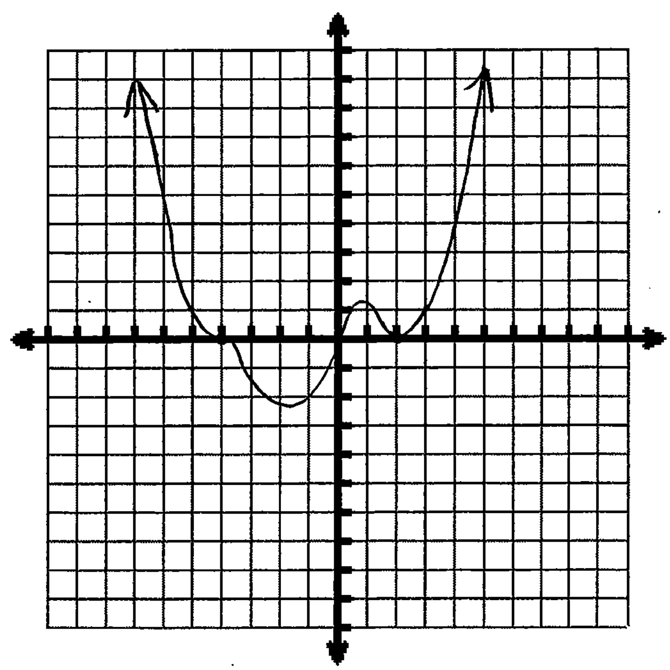
20. $f(x) = x(x-2)^2(x+4)^3$

$0 = x(x-2)^2(x+4)^3$

x-intercepts: 0
2 mult. 2
-4 mult. 3

y-intercept: (0, 0)

End Behavior: ↑ ↑



21. Graph by finding x-intercepts, y-intercepts, end behavior and multiplicities.

$f(x) = x^6 - 4x^5 - 5x^4$

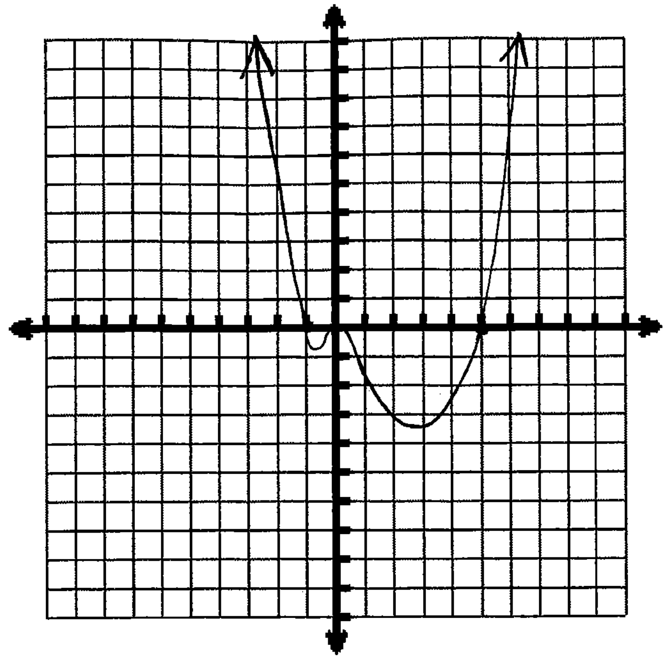
$0 = x^4(x^2 - 4x - 5)$

$0 = x^4(x-5)(x+1)$

x-intercepts: 0 mult. 4
5
−1

y-intercept: (0, 0)

End Behavior: ↑↑



22. Using the equation $g(x) = x^5 - 144x^3$, list combinations of possible types of solutions.

5 real and 0 imaginary
3 real and 2 imaginary
1 real and 4 imaginary

23. Complete the rational root theorem ~~to completely~~ solve this equation.

$h(x) = x^3 + 4x^2 - 49x - 66$

a. List the possible roots.

$\dfrac{\pm 1 \pm 2 \pm 3 \pm 6 \pm 11 \pm 22 \pm 33 \pm 66}{\pm 1} = \boxed{\pm 1, \pm 2, \pm 3, \pm 6, \pm 11, \pm 22, \pm 33, \pm 66}$

b. Use any method to find the one that works and test it here.

$$\begin{array}{r|rrrr} 6 & 1 & 4 & -49 & -66 \\ & & 6 & 60 & 66 \\ \hline & 1 & 10 & 11 & 0 \end{array}$$

$\boxed{x = 6}$

c. Use the equation left over in part b and solve it.

$x^2 + 10x + 11 = 0$

$x = \dfrac{-10 \pm \sqrt{(10)^2 - 4(1)(11)}}{2(1)} = \dfrac{-10 \pm \sqrt{100-44}}{2} = \dfrac{-10 \pm \sqrt{56}}{2} = \dfrac{-10 \pm \sqrt{4}\sqrt{14}}{2} = \dfrac{-10 \pm 2\sqrt{14}}{2} = \boxed{-5 \pm \sqrt{14}}$

d. Write all 3 solutions here, $\boxed{6}$, $\boxed{-5+\sqrt{14}}$, & $\boxed{-5-\sqrt{14}}$.

You should also study your 6.1 – 6.3 Quiz and 6.5 Mini Quiz!