Algebra 2B Final Exam Review
March 2016

Name: Key Date: _____ Hour: ____

Chapter 8

1) Determine the value of each of the following.

a. $\log 1000 = 3$ b. $\log_2 \frac{1}{32} = -5$ c. $\log_{36} 6 = \frac{1}{2}$ d. $\log_{\frac{1}{3}} 27 = -3$

2) Rewrite each logarithmic equation as an exponential equation and vice versa.

a. $2^3 = 8$ $\log_2 8 = 3$

b. $e^4 = 54.6$ $\ln 54.6 = 4$

c. $\log_{125} 5 = \frac{1}{3}$ $125^{\frac{1}{3}} = 5$

d. $\ln e = 1$ $e^1 = e$

3) Write each logarithmic expression as a single logarithm.

a. $5\log x + 3\log x^2$

$= \log x^5 + \log x^6 = \log(x^5 * x^6)$

$= \log x^{11}$

b. $\log_3 2x - 5\log_3 y$

$= \log_3 2x - \log_3 y^5$

$= \log_3 \left[\frac{2x}{y^5}\right]$

c. $\log_5 y - 2(\log_5 x + 2\log_5 z)$

$= \log_5 y - 2(\log_5 x + \log_5 z^2)$

$= \log_5 y - 2(\log_5 xz^2)$

$= \log_5 y - \log_5 x^2z^4 = \log_5 \left[\frac{y}{x^2z^4}\right]$

d. $\frac{1}{2}\ln 4x + \frac{1}{2}\ln 9y - 2\ln z$

$= \frac{1}{2}\ln 36xy - 2\ln z$

$= \ln \sqrt{36xy} - \ln z^2$

$= \ln \left[\frac{6\sqrt{xy}}{z^2}\right]$

4) Solve the following equations for x. Round your answer to the nearest thousandth.

a. $e^{2x-1} = 20$

$\ln 20 = 2x - 1$

$+1 \quad +1$

$\frac{\ln 20 + 1}{2} = \frac{2x}{2}$

$x \approx 1.998$

b. $e^{3x} = 10$

$\frac{\ln 10}{3} = \frac{3x}{3}$

$x \approx .768$

c. $13^{x+2} - 10 = 186$

$+10 \quad +10$

$13^{x+2} = 196$

$\log_{13} 196 = x + 2$

$-2 \quad -2$

$x \approx .058$

d. $2^{3x} + 2 = 10$

$-2 \quad -2$

$2^{3x} = 8$

$\log_2 8 = 3x$

$3 = 3x$

$x = 1$

e. $2\ln x - \ln 5 = -10$

$\ln x^2 - \ln 5 = -10$

$\ln \frac{x^2}{5} = -10$

$\frac{x^2}{5} = e^{-10}$

$x^2 = 5e^{-10}$

$x = \sqrt{5e^{-10}}$

$x \approx .015$

f. $\log(x-15) + \log x = 2$

$\log[x(x-15)] = 2$

$x(x-15) = 10^2$

$x(x-15) = 100$

$x^2 - 15x - 100 = 0$

$(x-20)(x+5) = 0$

$x = 20$

$x = -5$

5) Expand each logarithm.

a. $\log_4 5x^2 = \log_4 5 + \log_4 x^2$

$= \log_4 5 + 2\log_4 x$

b. $\log_6(2xyz)^3 = \log_6 2^3x^3y^3z^3$

$= \log_6 2^3 + \log_6 x^3 + \log_6 y^3 + \log_6 z^3$

$= 3\log_6 2 + 3\log_6 x + 3\log_6 y + 3\log_6 z$

c. $\ln\frac{2x^2y}{3k^3} = \ln 2x^2y - \ln 3k^3$

$= \ln 2 + \ln x^2 + \ln y - \ln 3 - \ln k^3$

$= \ln 2 + 2\ln x + \ln y - \ln 3 - 3\ln k$

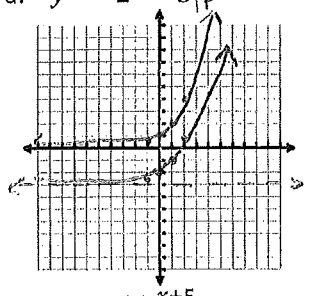
d. $\log\sqrt{\frac{2rst}{5w}} = \log\frac{2^{\frac{1}{2}}r^{\frac{1}{2}}s^{\frac{1}{2}}t^{\frac{1}{2}}}{5^{\frac{1}{2}}w^{\frac{1}{2}}}$

$= \log 2^{\frac{1}{2}} + \log r^{\frac{1}{2}} + \log s^{\frac{1}{2}} + \log t^{\frac{1}{2}} - \log 5^{\frac{1}{2}} - \log w^{\frac{1}{2}}$

$= \frac{1}{2}\log 2 + \frac{1}{2}\log r + \frac{1}{2}\log s + \frac{1}{2}\log t - \frac{1}{2}\log 5 - \frac{1}{2}\log w$

6) Graph the following equations. Label a minimum of 3 points and any asymptotes. State the domain and range.

a. $y = 2^x - 3$



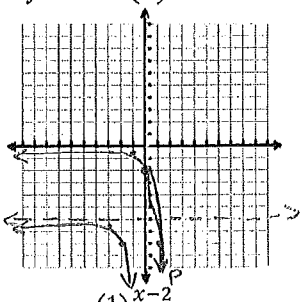
parent $y = 2^x$

x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

D: $\mathbb{R}$

R: $(-3,\infty)$

b. $y = -2(4)^{x+2} - 6$



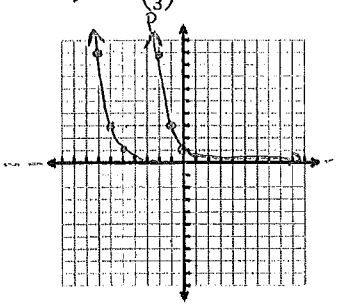
parent $y = -2 * 4^x$

x	y
-1	$-\frac{1}{2}$
0	-2
1	-8
2	-32

D: $\mathbb{R}$

R: $(-\infty,-6)$

c. $y = \left(\frac{1}{3}\right)^{x+5}$



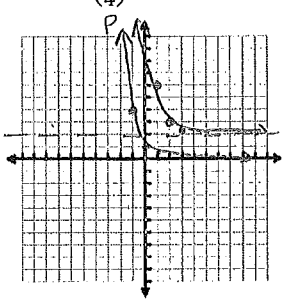
parent $y = \left(\frac{1}{3}\right)^x$

x	y
-2	9
-1	3
0	1
1	$\frac{1}{3}$

D: $\mathbb{R}$

R: $(0,\infty)$

d. $y = \left(\frac{1}{4}\right)^{x-2} + 2$



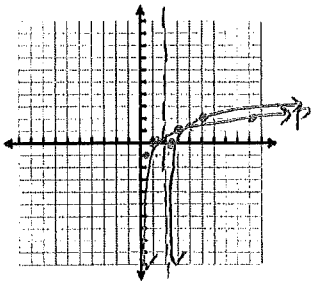
parent $y = \left(\frac{1}{4}\right)^x$

x	y
-2	16
-1	4
0	1
1	$\frac{1}{4}$

D: $\mathbb{R}$

R: $(2,\infty)$

e. $y = \log_3(x-2) + 1$



parent $y = \log_3 x$

x	y
$\frac{1}{3}$	-1
1	0
3	1
9	2

D: $(2,\infty)$

R: $\mathbb{R}$

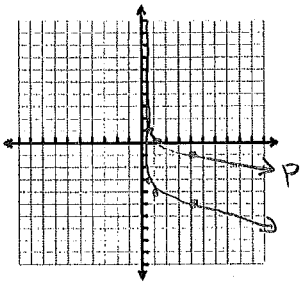
$3^{-1} = \frac{1}{3}$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

f. $y = -\log_4 x - 4$



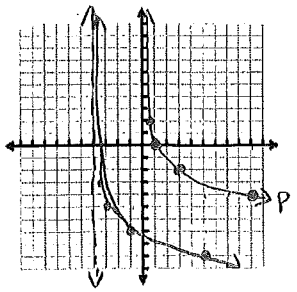
parent $y = -\log_4 x$

x	y
$\frac{1}{4}$	1
1	0
4	-1

D: $(0,\infty)$

R: $\mathbb{R}$

g. $y = -2\log_3(x+4) - 5$



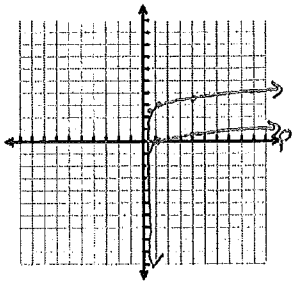
parent $y = -2\log_3 x$

x	y
$\frac{1}{3}$	2
1	0
3	-2
9	-4

D: $(-4,\infty)$

R: $\mathbb{R}$

h. $y = \frac{1}{2}\log_4 x + 3$



parent $y = \frac{1}{2}\log_4 x$

x	y
$\frac{1}{4}$	$-\frac{1}{2}$
1	0
4	$\frac{1}{2}$

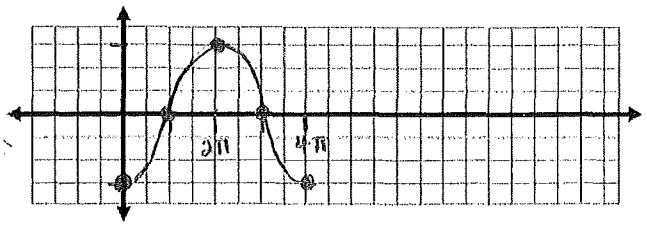
D: $(0,\infty)$

R: $\mathbb{R}$

Chapter 13

For problems #1, 3, and 5, graph one complete cycle. For problems #2, 4, and 6, graph two complete cycles. Then, state the period, the amplitude, and the range in the spaces provided.

1) $y = -3\cos\left(\frac{\theta}{2}\right)$

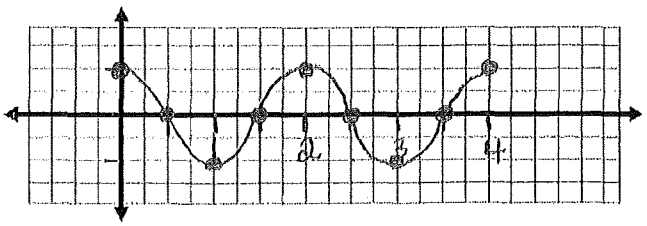


Period: $4\pi = \frac{2\pi}{\frac{1}{2}}$

Amplitude: 3

Range: $[-3, 3]$

2) $y = 2\cos\pi\theta$

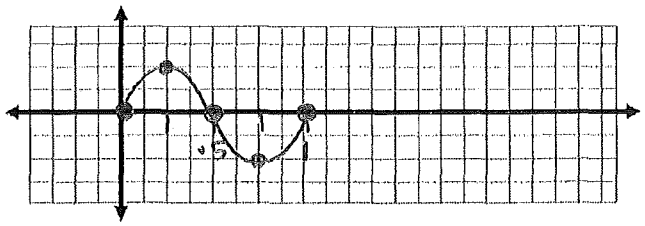


Period: $2 = \frac{2\pi}{\pi}$

Amplitude: 2

Range: $[-2, 2]$

3) $y = 2\sin 2\pi\theta$

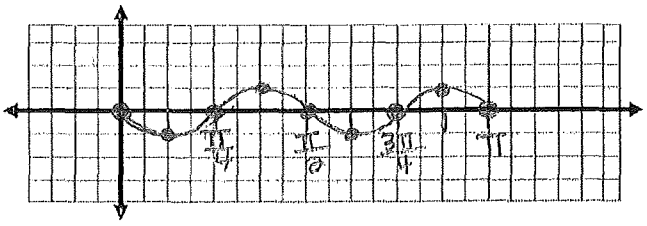


Period: $1 = \frac{2\pi}{2\pi}$

Amplitude: 2

Range: $[-2, 2]$

4) $y = -\sin(4\theta)$

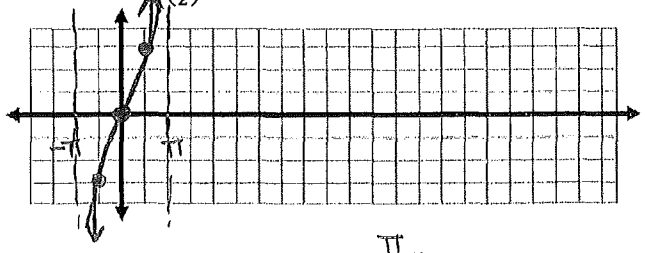


Period: $\frac{\pi}{2} = \frac{2\pi}{4}$

Amplitude: 1

Range: $[-1, 1]$

5) $y = 3\tan\left(\frac{\theta}{2}\right)$

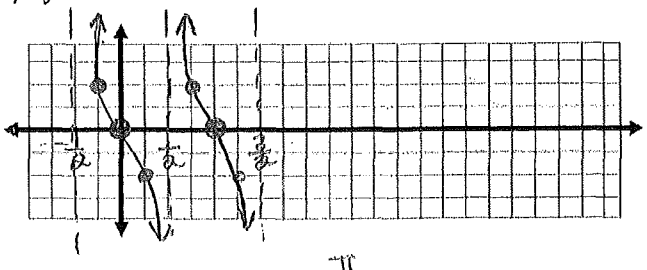


Period: $2\pi = \frac{\pi}{\frac{1}{2}}$

Range: $\mathbb{R}$

$x = \pm\frac{2\pi}{2} = \pm\pi$

6) $y = -2\tan\pi\theta$



Period: $1 = \frac{\pi}{\pi}$

Range: $\mathbb{R}$

$x = \pm\frac{1}{2}$

7) Jeremy bought a new car at a cost of $22,000. The car depreciates 10% each year. How much will it be worth in 10 years? When will it be worth $10,000?

$y = 22000(.9)^x$

$= 22000(.9)^{10}$

$\approx \$7670.93$ after 10 yrs

$\frac{10000}{22000} = \frac{22000(.9)^x}{22000}$

$\left(\frac{10000}{22000}\right) = (.9)^x$

$\log_{.9}\left(\frac{10000}{22000}\right) = x$

$x \approx 7.48$ yrs.

8) The number of wolves in the wild in the northern section of the Cattaraugus County is decreasing exponentially. Five years ago, an environmental studies class counted 112 wolves in the area, and this year's environmental studies class has counted 80 wolves in the area. After how many years will this population drop below 15 wolves if this rate of decrease continues?

2011 2016

$(0,112) \rightarrow \boxed{a = 112}$

$(5,80)$

$y = a * b^x$

$80 = 112 * b^5$

$\frac{80}{112} = b^5$

$b = \sqrt[5]{\frac{80}{112}}$

$\boxed{b \approx .935}$

$y = 112(.935)^x$

$\frac{15}{112} = \frac{112(.935)^x}{112}$

$\frac{15}{112} = (.935)^x$

$\log_{.935}\left(\frac{15}{112}\right) = x$

$x \approx 29.91$ yrs

9) A scientist is growing bacteria in a Petri dish. After 2 hours there are 2030 bacteria in the dish and after 5 hours there are 10240 bacteria in the dish.

$(2, 2030)$ & $(5, 10240)$

$y = a * b^x$

$2030 = a * b^2$

$\frac{2030}{b^2} = a$

$10240 = a * b^5$

$10240 = \frac{2030}{b^2} b^5$

$\frac{10240}{2030} = \frac{2030}{2030} b^3$

$\frac{10240}{2030} = b^3$

$\sqrt[3]{\frac{10240}{2030}} = b$

a. What is the growth rate of the bacteria?

$\boxed{b = 1.715}$ $\boxed{\text{Growth rate} = 71.5\%}$

b. How many bacteria were in the dish at the beginning?

$\frac{2030}{(1.715)^2} = a$

$\boxed{a = 690}$

c. How many bacteria will be in the dish after 10 hours?

$y = 690 * 1.715^x$

$= 690 * 1.715^{10} \approx 151890$ bacteria

10) Strontium-90 has a half-life of 25 years. How long would it take for 40 mg of it to decay to: $y = 40 * \left(\frac{1}{2}\right)^{\frac{x}{25}}$

a. 20 mg

$20 = 40\left(\frac{1}{2}\right)^{\frac{x}{25}}$

$\frac{1}{2} = \frac{1}{2}^{\frac{x}{25}} \rightarrow x = 25$ years

b. 1.25 mg

$\frac{1.25}{40} = \frac{40}{40}\left(\frac{1}{2}\right)^{\frac{x}{25}}$

$(.03125) = \left(\frac{1}{2}\right)^{\frac{x}{25}}$

$25 * \log_{\frac{1}{2}} .03125 = \frac{x}{25} * 25$

$x = 125$ years

11) A bacteria culture triples every 4 hours and starts with 10,000 bacteria. Find the number of bacteria in the culture after 30 hours.

$y = 10000 * 3^{\frac{x}{4}}$

$= 10000 * 3^{\frac{30}{4}}$

≈ 378719952 bacteria

7) Which angle, in standard position, is NOT coterminal with the others?

a. $-130°$ $+360$ 230 b. $230°$ c. $50°$ d. $590°$

8) Convert each angle measure in to its equivalent in radians or degrees.

a. $\frac{-220° * \pi}{180} = \frac{-11\pi}{9}$ b. $\frac{40° \cdot \pi}{180} = \frac{2\pi}{9}$ c. $\frac{-5\pi}{3}$ radians $\frac{-5(180)}{3} = -300$ d. $\frac{5\pi}{6}$ radians $\frac{5(180)}{6} = 150$

9) Find the measure of an angle between 0° and 360° coterminal with each given angle.

a. $-140°$ $+360$ $220°$ b. $570°$ -360 $210°$ c. $-760°$ $+360$ $+360$ $+360$ $320°$

10) Find the measure of an angle between 0 and 2π coterminal with each given angle.

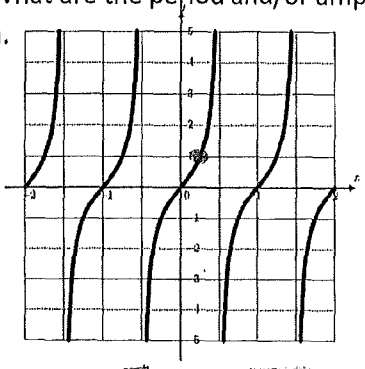
a. $\frac{17\pi}{6} - 2\pi = \frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$ b. $-5\pi + 2\pi + 2\pi + 2\pi = \pi$ c. $\frac{21\pi}{4} - 2\pi = \frac{21\pi}{4} - \frac{8\pi}{4} = \frac{13\pi}{4} - \frac{8\pi}{4} = \frac{5\pi}{4}$

11) A gear with a radius of 5 in. turns through an angle of 330°. How far does a point on the edge of the gear travel as the gear turns through this angle?

$330° \rightarrow \frac{330 * \pi}{180} = \frac{11\pi}{6}$ $s = r\theta = 5\left(\frac{11\pi}{6}\right) = \frac{55\pi}{6}$ in

$s = \frac{\theta}{360} \cdot 2\pi r = \frac{330}{360} \cdot 2\pi(5) = \frac{11}{12} \cdot 10\pi = \frac{55\pi}{6}$ in

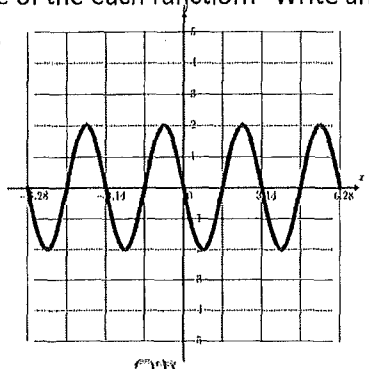
12) What are the period and/or amplitude of the each function? Write an equation for each graph.

a. 

$1 = \frac{\pi}{b}$ $b = \pi$

Period: 1

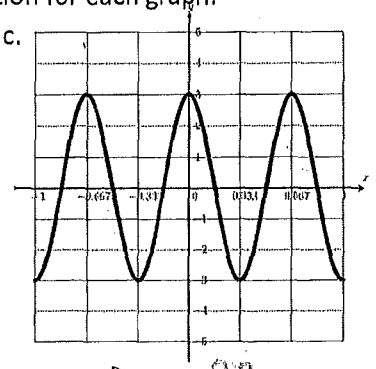
Equation: $y = \tan \pi x$

b. 

$\pi = \frac{2\pi}{b} = 2$

Period: π Amplitude: 2

Equation: $y = -2\sin 2x$

c. 

$\frac{2}{3} = \frac{2\pi}{b}$ $b = 3\pi$

Period: $\frac{2}{3}$ Amplitude: 3

Equation: $y = 3\cos 3\pi x$

13) On your unit circle, label the indicated angles by their degree measure, radian measure, and coordinates. Then, use it to answer the following questions:

a. $\sin 150° = \frac{1}{2}$

b. $\cos -60° = \frac{1}{2}$

c. $\tan 225° = 1$

d. $\sin -300° = \frac{\sqrt{3}}{2}$

e. $\cos -90° = 0$

f. $\tan 180° = 0$

g. $\sin 4\pi = 0$

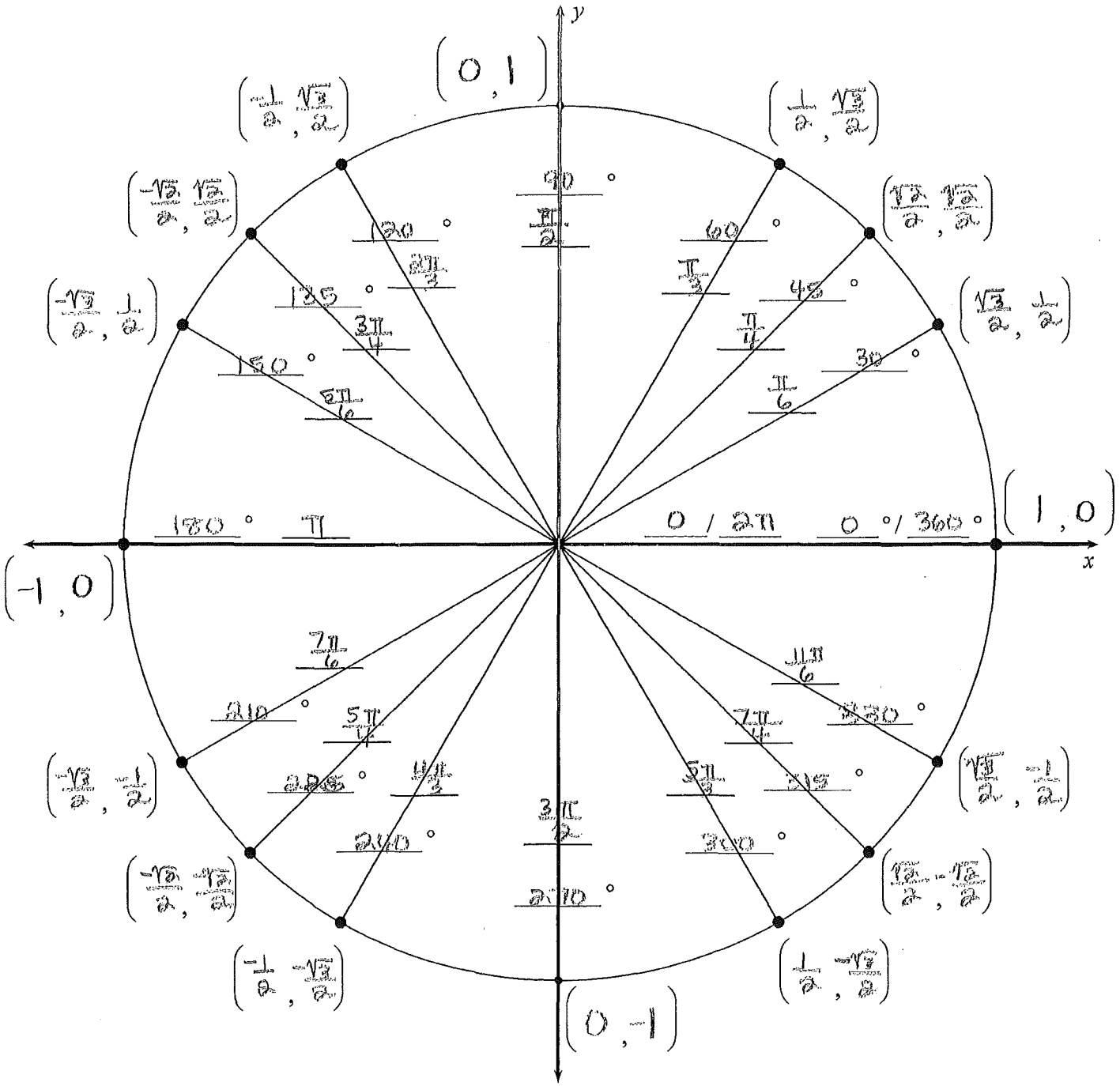
h. $\cos \frac{5\pi}{3} = \frac{1}{2}$

i. $\tan \frac{3\pi}{2} =$ undefined

j. $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

k. $\cos \frac{-10\pi}{3} = -\frac{1}{2}$

l. $\tan \frac{4\pi}{3} = \sqrt{3}$



Eureka

1) Only 93% of the airplane parts David is examining pass inspection. What is the probability that all of the next five parts pass inspection?

$P(5 \text{ in a row pass}) = (.93)^5 = .6957$

2) High school students in one school chose their favorite leisure activity. Find each probability. Round to the nearest tenth of a percent.

Favorite Leisure Activities

	Sports	Hiking	Reading	Phoning	Shopping	Other	
Female	39	48	85	62	71	29	334
Male	67	58	76	54	68	39	362
	106	106	161	116	139	68	696

a. $P(sports\ given\ female) = \frac{39}{334} = 11.7\%$

b. $P(female\ given\ sports) = \frac{39}{106} = 36.8\%$

c. $P(reading\ given\ male) = \frac{76}{362} = 21.0\%$

d. $P(male\ given\ reading) = \frac{76}{161} = 47.2\%$

e. $P(hiking\ given\ female) = \frac{48}{334} = 14.4\%$

f. $P(hiking\ given\ male) = \frac{58}{362} = 16.0\%$

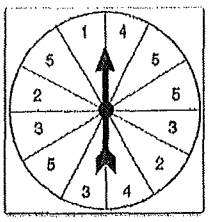
g. $P(male\ given\ shopping) = \frac{68}{139} = 48.9\%$

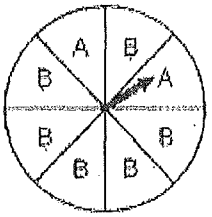
h. $P(female\ given\ shopping) = \frac{71}{139} = 51.1\%$

i. $P(female\ given\ not\ phoning) = \frac{272}{580} \approx 46.9\%$

j. $P(\text{not } \overset{\text{male}}{female}\ given\ phoning) = \frac{54}{116} = 46.6\%$

3) The diagram to the right shows two spinners. For the first spinner, the contestant can spin a 1, 2, 3, 4, or 5 and for the second spinner, the contestant can spin an A or a B. Each region is equally likely, but you will notice that each outcome is not equally likely. Both spinners will be spun. Writing your answers as fractions in lowest terms, find the probability of each event.





a. The contestant spins a 3 and a B. $P(3 \text{ and } B) = P(3)\,P(B) = \frac{3}{12} * \frac{6}{8} = \frac{18}{96} = \frac{3}{16}$

b. The contestant spins a B on six consecutive spins.
$P(B \text{ on six consecutive turns}) = \left(\frac{6}{8}\right)^6 = \frac{729}{4096}$

c. After three turns, the sum of the contestant's spins on the number spinner is less than 4.
$P(\text{less than } 4) = P(1,1,1) = \left(\frac{1}{12}\right)\left(\frac{1}{12}\right)\left(\frac{1}{12}\right) = \frac{1}{1728} \approx 0.058\%$

d. After three turns, the sum of the contestant's spins on the number spinner is at least 4.
$P(\text{at least } 4) = 1 - P(\text{less than } 4) = 1 - \frac{1}{1728} = \frac{1727}{1728} \approx 99.94\%$

4) The senior class is 55% female, and 32% are females who play a competitive sport. Find the probability that a student plays a competitive sport, given that the student is female.

$P(F) = .55$

$P(F \text{ and } CS) = .32$

$P(CS \text{ given } F) = \frac{P(F \text{ and } CS)}{P(F)}$

$= \frac{.32}{.55} = .58$

5) You want to see if fifth-grade boys or fifth-grade girls are faster at solving Sudoku puzzles. You randomly select twenty fifth-grade boys and twenty fifth-grade girls from fifth graders in your school district. You time and record how long it takes each student to solve the same Sudoku puzzle correctly.

a. What type of a statistical study is being described here? observational study

b. The mean time of the twenty fifth-grade boys was 2 minutes and 7 seconds. Is this a sample statistic or a population characteristic? Explain.

sample statistic because this data was calculated from how the sample performed

6) A call center for Target receives many types of calls. The probability that a call is about store hours is 0.50 and the probability that a call is about the availability of a product is 0.20. $P(S) = .5$ $P(P) = .2$

a. Suppose that you are told that the probability that a call is about stores hours and product availability is 0.15. $P(S \text{ and } P)$ Calculate the probability that a randomly selected call is about store hours or product availability.

$P(S \text{ or } P) = P(S) + P(P) - P(S \text{ and } P)$

$= .5 + .2 - .15 = .55$

b. Suppose now that you are not given the information from part (a), but you are told that the events "the call is about store hours" and "the call is about product availability" are independent. Now calculate the probability that a randomly selected call is about store hours or product availability.

$P(S \text{ and } P) = P(S)\,P(P)$

$= .5 * .2 = .1$

$P(S \text{ or } P) = .5 + .2 - .1 = .60$

7) Leonard Loves-His-Phone is always on his iPhone. The probability that he updates his Snapchat Story every hour is 0.9. The probability that he tweets every hour is 0.7. The probability that he checks Instagram every hour is 0.4.

a. What is the probability that Leonard has updated his Snapchat Story, has tweeted, and has checked Instagram this hour?

$P(SS, T, \text{and } I) = .9 * .7 * .4 = .252$

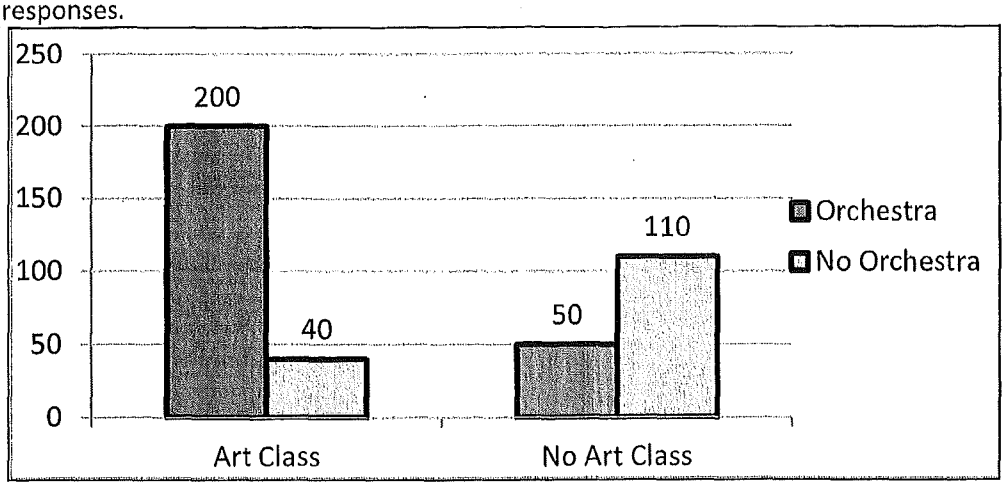
b. What is the probability that Leonard has not updated his Snapchat Story, has not tweeted, and has not checked Instagram this hour?

$P(\text{not } SS, \text{not } T, \text{and not } I) = .1 * .3 * .6 = .018$

c. What is the probability that Leonard has updated his Snapchat Story, has tweeted, and has not checked Instagram this hour?

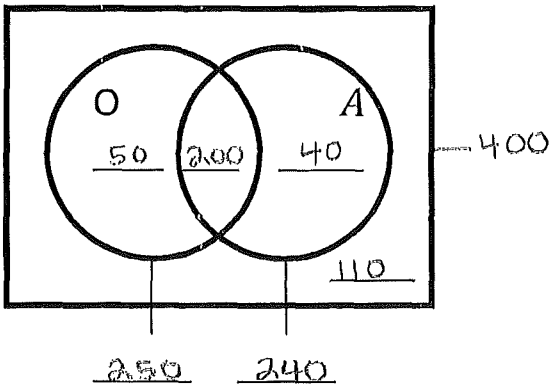
$P(SS, T, \text{and not } I) = .9 * .7 * .6 = .378$

8) Students at Bayside High School were asked about their extracurricular classes. The bar graph below shows their responses.

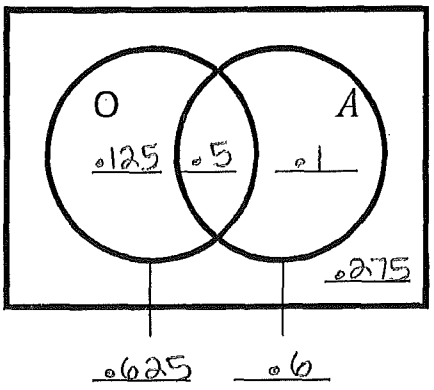


a. Based on the graph above, complete the following Venn diagrams where O is the event that a student is in orchestra and A is the event that a student is taking an art class. You should fill in 6 values for each Venn diagram on the lines provided.

i. Complete with number of students.



ii. Complete with probabilities of each event.



b. What is the probability that a randomly selected student is not taking an art class?

$\frac{160}{400} = .4$

c. If a randomly selected student is taking an art class, what is the probability that they are not in orchestra?

$\frac{40}{240} = .17$

d. Calculate the following probabilities:

i. $P(A \cap O) = .5$

ii. $P(A \text{ given } O) = \frac{200}{250} = .8$

iii. $P(A \text{ or } O) = .125 + .5 + .1 = .725$

iv. $P(O \text{ and not } A) = .125$

v. $P(O \text{ given } A) = \frac{200}{240} = .83$

vi. $P(A \text{ or } A^C) = 1$

9) The students of a high school are 51% males; 45% of the males and 49% of the females attend concerts. Fill in the following hypothetical two-way table using the given information.

	Attends concerts	Doesn't attend concerts	Total
Male	2295	2805	5100
Female	2401	2499	4900
Total	4696	5304	10000

a. Find the probability that a student attends concerts.

$\frac{4696}{10000} = 46.96\%$

b. Find the probability that a student is a female and does not attend concerts.

$\frac{2499}{10000} = 24.99\%$

c. Are the events "attends concerts" and "male" independent? Explain your reasoning.

$P(\text{male}) = .51$

$P(\text{male given attends concerts}) = \frac{2295}{4696} = .49$

not independent
$.51 \neq .49$

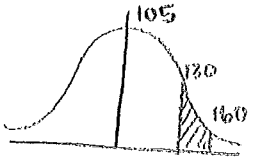
10) In one area the probability of a power outage during a rainstorm is 4%. Find *P*(at least 1 outage in the next 5 rainstorms).

$P(\text{at least on outage in next 5 showers}) = 1 - P(\text{not outages})$

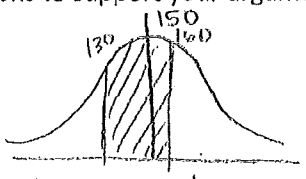
$= 1 - (.96)(.96)(.96)(.96)(.96)$

$= .185$

11) The durations of movies are approximately normally distributed. Hollywood movies have a mean duration of 105 minutes and a standard deviation of 15 minutes. Bollywood movies have a mean duration of 150 minutes and a standard deviation of 15 minutes. Is it more likely that a Hollywood movie will last between 130 and 160 minutes or that a Bollywood movie will last between 130 and 160 minutes? Answer this question without doing any calculations. Include graphs to support your argument.



Hollywood



Bollywood

It is more likely that Bollywood movies will last between 130 and 160 minutes because the shaded ave is larger. Also 130 is below the mean while 160 is above where as for Hollywood movies 130 and 160 are both above the mean.

12) The average time in the men's 100-meter sprint in the 2012 Olympics was 10.10 seconds and the standard deviation was 0.77 seconds. Assuming the values were normally distributed, find the following probabilities.

a. What is the probability that a runner ran the race in less than 9 seconds?

normalcdf(-1000, 9, 10.10, .77) $\approx$ .0766

b. What is the probability that a runner ran the race in more than 9 seconds?

$1 - P(< 9) = .9234$

c. What is the probability that a runner ran the race between 7 seconds to 11 seconds?

normalcdf(7, 11, 10.10, .77) = .8787

d. What is the probability that a runner ran the race in more than 12 seconds?

normalcdf(12, 1000, 10.10, .77) = .0068