Algebra 2 3Tri B
Final Exam Review (Chapters 6, 7, & 9)
Winter 2018

Name: Key
Date: _____ Hour: _____

Chapter 6 - Polynomials

1. Divide using long division and/or synthetic division.

a. $(x^3+3x^2-2x-4)\div(x-2)$

$$\begin{array}{r|cccc} 2 & 1 & 3 & -2 & -4 \\ & & 2 & 10 & 16 \\ \hline & 1 & 5 & 8 & 12 \end{array}$$

x^2+5x+8, rem 12

b. $(x^4-x+2)\div(x^2+1)$

$$\begin{array}{r} x^2 \quad -1 \\ x^2+1 \overline{)\,x^4+0x^3+0x^2-x+2} \\ -(x^4 \quad +x^2) \\ \hline -x^2-x+2 \\ -(-x^2 \quad -1) \\ \hline -x+3 \end{array}$$

$x^2-1+\frac{-x+3}{x^2+1}$

2. Determine the end behavior of the graphs of these functions.

a. $y=3x^4-5x^2+3x-4$

deg = even
lead coeff > 0
up, up

b. $y=-5x^3-3x+1$

deg = odd
lead coeff < 0
up, down

3. Write an equation in standard form of a second degree polynomial that has zeros of 7 and -2.

$f(x) = (x-7)(x+2)$

$f(x) = x^2-5x-14$

4. Write an equation in standard form of a third degree polynomial that has zeros of 5 and $3i$.

$f(x) = (x-5)(x-3i)(x+3i)$
$= (x-5)(x^2-9i^2)$
$= (x-5)(x^2+9)$
$f(x) = x^3-5x^2+9x-45$

5. A polynomial equation has roots 3, -4i and 2 – 3i, what is the least possible degree of the polynomial?

$3, -4i, 4i, 2-3i, 2+3i \rightarrow$ at least 5th degree

6. Test to see if -3 is a possible root of $6x^3 - 4x^2 + x - 3 = 0$.

$$\begin{array}{r|cccc} -3 & 6 & -4 & 1 & -3 \\ & & -18 & 66 & -201 \\ \hline & 6 & -22 & 67 & -204 \end{array}$$

not zero. Thus -3 is not a root.

7. Is x + 1 a factor of $f(x) = 5x^4 - 8x^2 + 15x - 10$?

$$\begin{array}{r|ccccc} -1 & 5 & 0 & -8 & 15 & -10 \\ & & -5 & 5 & 3 & -18 \\ \hline & 5 & -5 & -3 & 18 & -28 \end{array}$$

not zero

no

8. Find p(4) if $p(x) = -3x^3 - 7x^2 + 2x + 12$ (without the use of a calculator)

$$\begin{array}{r|cccc} 4 & -3 & -7 & 2 & 12 \\ & & -12 & -76 & -296 \\ \hline & -3 & -19 & -74 & -284 \end{array}$$

$P(4) = -284$

9. Solve for x by factoring.

a. $x^4 - 3x^2 - 18 = 0$

$(x^2 - 6)(x^2 + 3) = 0$

$x^2 - 6 = 0$ $\quad$ $x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{6}$ $\quad$ $\sqrt{x^2} = \sqrt{-3}$

$x = \pm\sqrt{6}$ $\quad$ $x = \pm i\sqrt{3}$

10. For the following polynomial ~~$5x^3 - 2x^2 + x - 15 = 0$~~ → $2x^3 + 2x^2 - 8x - 8 = 0$

a. List all the possible rational roots (Rational Root Theorem)

~~factors of 15 / factors of 5 = ±1, ±3, ±5, ±15 / ±1, ±5 = ±1, ±3, ±5, ±15, ±3/5~~

b. Graph to find one that works and test it with synthetic division.

~~★ homework irrational roots~~

p = 8 | 8, 1, 4, 2

q = 2 | 1, 2

±4, ±8, ±1/2, ±1, ±2

$$\begin{array}{r|rrrr} 2 & 2 & 2 & -8 & -8 \\ & & 4 & 12 & 8 \\ \hline & 2 & 6 & 4 & 0 \end{array}$$

c. Use part b to help find the rest of the solutions (there should be 3!)

~~(1.53859)~~

$2x^2 + 6x + 4 = 0$

$2(x^2 + 3x + 2) = 0$

$2(x + 2)(x + 1) = 0$

$x = 2$

$x = -2, -1$

11. For each function, find the following and then graph. You may have to simplify the first one. (without a calculator)

f(x) = $x^4 + 2x^2 - 8$ = $(x^2 + 4)(x^2 - 2)$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

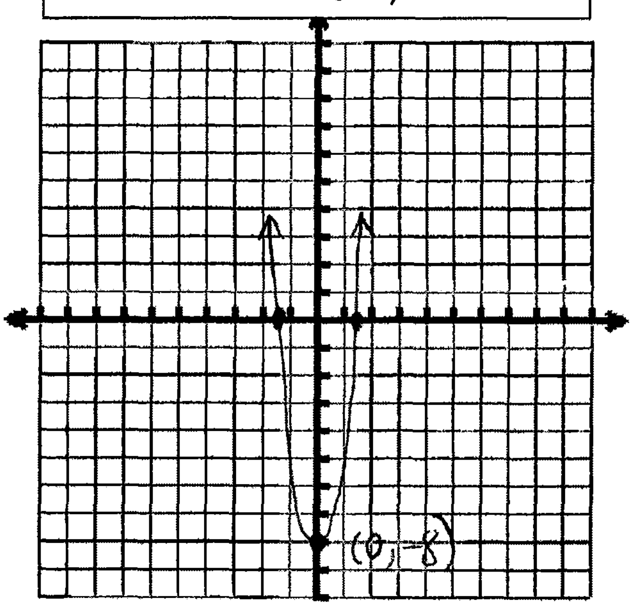
Domain: ℝ

Range: $y \geq -8$

x-intercept(s) with multiplicities:

$\pm\sqrt{2}$ mult = 1

y-intercept: ~~(0, 8)~~ (0, −8)



g(x) = $x(x-1)^2(x+4)$

deg = 4, LC > 0

up, up

Domain: ℝ

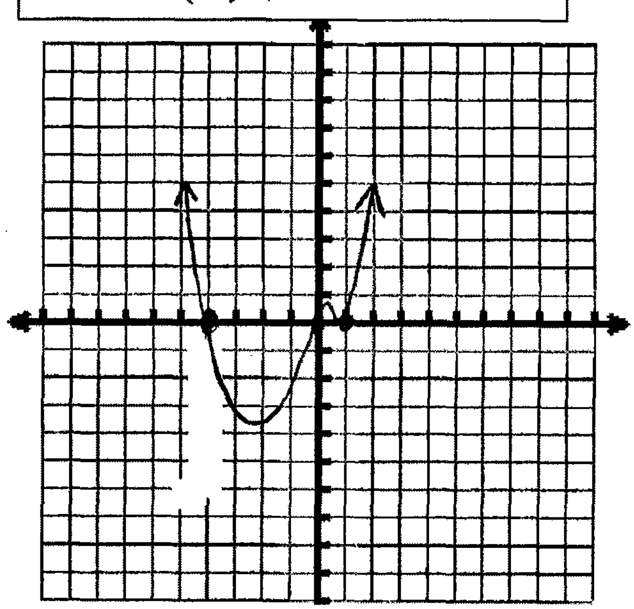
Range: ℝ

x-intercept(s) with multiplicities:

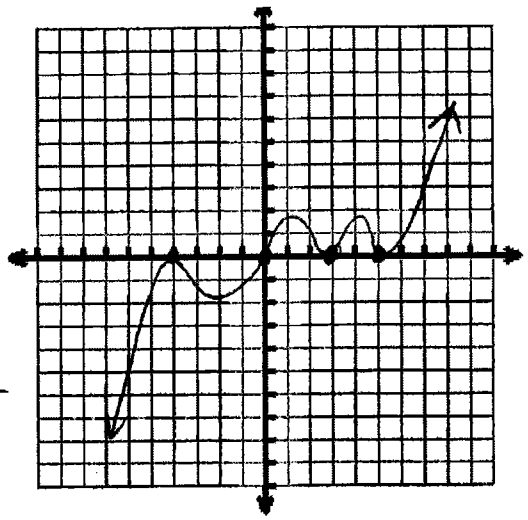
x = 0, x = 1, x = −4

mult = 0, mult = 2, mult = 1

y-intercept: (0, 0)



12. Graph a 7th degree polynomial function with a positive leading coefficient given its zeros are 3 with a multiplicity of 2, 0 with a multiplicity of 1, 5 with a multiplicity of 2, and -4 with a multiplicity of 2.



deg = odd → down up
LC > 0

$x=3$, mult = 2 → bounce
$x=0$, mult = 1 → cross
$x=5$, mult = 2 → bounce
$x=-4$, mult = 2 bounce

13. Describe the combinations of possible number and types of roots of the equation $x^4 - 4x^2 = 0$.

4th degree: ① 0 real, 4 imaginary
② 2 real, 2 imaginary
③ 4 real, 0 imaginary

Chapter 7 – Radicals

14. Let $f(x) = x^2 + 4$. Find:

a. $f^{-1}(x)$

$x = y^2 + 4$
$\sqrt{x-4} = \sqrt{y^2}$
$y = \pm\sqrt{x-4}$
$f^{-1}(x) = \pm\sqrt{x-4}$

b. $(f^{-1} \circ f)(5)$ **5**

c. $(f \circ f^{-1})(c)$ **C**

For #15 – 20, let f(x) = 2x + 5 and g(x) = x^2 – 3x + 2

15. f(7) + g(3)

$(2(7)+5) + (3^2 - 3(3) + 2)$
19 + 2
21

16. 4f(7) + 6g(3)

4(19) + 6(2)
76 + 12
88

17. f(x) – g(x)

$(2x+5) - (x^2 - 3x + 2)$
$2x + 5 - x^2 + 3x - 2$
$-x^2 + 5x + 3$

18. (g + f)(x)

$= g(x) + f(x)$
$(x^2 - 3x + 2) + (2x + 5)$
$x^2 - x + 7$

19. $(f \circ g)(x)$

$f(g(x))$
$f(x^2 - 3x + 2)$
$2(x^2 - 3x + 2) + 5$
$2x^2 - 6x + 9$

20. g(f(x))

$g(2x+5)$
$(2x+5)^2 - 3(2x+5) + 2$
$4x^2 + 20x + 25 - 6x - 15 + 2$
$4x^2 + 14x + 12$

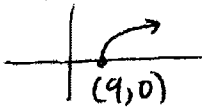
21. Describe how the graph $y = \sqrt{x-4} - 2$ is translated from the graph of $= \sqrt{x}$.

4 right, 2 down

22. Given $f(x) = \sqrt{x-9}$, state the **domain and range** of f(x), then find **the inverse** of the function.

domain: $x - 9 \geq 0$
D: $x \geq 9$

range $y \geq 0$



$x = \sqrt{y-9}$
$x^2 = y - 9$
$x^2 + 9 = y$

$f^{-1}(x) = x^2 + 9$
$x \geq 0$

Solve. Check for extraneous solutions (without a calculator)

23. $\left(\sqrt{x-3}=x-5\right)^2$ square both sides

$x-3=x^2-10x+25$

$-x+3 \quad -x+3$

$0=x^2-11x+28$

$0=(x-7)(x-4)$

$\boxed{x=7}$, ~~4~~

24. $(3x+4)^{\frac{1}{3}\cdot 3}=(-5)^3$ cube both sides

$3x+4=-125$

$-4 \quad -4$

$\frac{3x}{3}=\frac{-129}{3}$

$\boxed{x=-43}$

25. $\left(\sqrt{x+4}=\sqrt{3x}\right)^2$

$x+4=3x$

$4=2x$

$\boxed{2=x}$

26. $\left[(x-4)^{\frac{1}{2}}=x-6\right]^2$

$x-4=x^2-12x+36$

$-x+4 \quad -x+4$

$0=x^2-13x+40$

$0=(x-8)(x-5)$

$\boxed{x=8}$, ~~5~~

Simplify. (without a calculator).

27. $\sqrt{7x^3}\cdot\sqrt{14x}$

$\sqrt{98x^4}$

$49\ 2$

$\boxed{7x^2\sqrt{2}}$

28. $\frac{\sqrt{7x^4y}}{\sqrt{63xy^2}}$ Simplify fraction first!

$\sqrt{\frac{x^3}{9y}}=\frac{\sqrt{x^2}}{3\sqrt{y}}\cdot\frac{\sqrt{y}}{\sqrt{y}}$

$=\boxed{\frac{\sqrt{x^3y}}{3y}}\rightarrow\boxed{\frac{x\sqrt{xy}}{3y}}$

29. $\sqrt{48}+2\sqrt{75}+5\sqrt{12}$

$\sqrt{3\cdot16}+2\sqrt{3\cdot25}+5\sqrt{3\cdot4}$

$4\sqrt{3}+10\sqrt{3}+10\sqrt{3}$

$\boxed{24\sqrt{3}}$

30. $(7+\sqrt{3})(3+5\sqrt{3})$

$21+35\sqrt{3}+3\sqrt{3}+5\sqrt{9}$

$21+38\sqrt{3}+5(3)$

$\boxed{36+38\sqrt{3}}$

31. $\frac{5}{3-2\sqrt{6}}\cdot\left(\frac{3+2\sqrt{6}}{3+2\sqrt{6}}\right)$

multiply by conjugate of the denominator.

$\frac{15+10\sqrt{6}}{9+6\sqrt{6}-6\sqrt{6}-4\sqrt{36}}$

$\frac{15+10\sqrt{6}}{9-24}=\frac{15+10\sqrt{6}}{-15}=\boxed{\frac{3+2\sqrt{6}}{-3}}$

32. $\frac{3+\sqrt{5}}{2-\sqrt{5}}\left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right)$

$\frac{6+3\sqrt{5}+2\sqrt{5}+5}{4-\sqrt{25}}$

$\frac{11+5\sqrt{5}}{-1}=\boxed{-11-5\sqrt{5}}$

33. $x^{\frac{1}{6}}\cdot x^{\frac{1}{3}}$ (add exponents)

$x^{\frac{1}{6}+\frac{2}{6}}$

$x^{\frac{3}{6}}=\boxed{x^{\frac{1}{2}}}$

34. $\left(\frac{8x^9y^3}{27x^2y^{12}}\right)^{\frac{2}{3}}$

$\left(\frac{8x^7}{27y^9}\right)^{\frac{2}{3}}=\boxed{\frac{4x^{14/3}}{9y^6}}$

35. $(125)^{-\frac{2}{3}}$

$\frac{1}{125^{2/3}}=\frac{1}{\left(\sqrt[3]{125}\right)^2}$

$\frac{1}{5^2}=\boxed{\frac{1}{25}}$

Continue simplifying (without a calc)

36. $\sqrt[3]{48}+5\sqrt[3]{24}$

$16\cdot 3 \quad 8\cdot 3$

$\sqrt[3]{2\cdot 2\cdot 2}\,2\cdot 3+5\sqrt[3]{2\cdot 2\cdot 2}\,3$

$2\sqrt[3]{6}+10\sqrt[3]{3}$

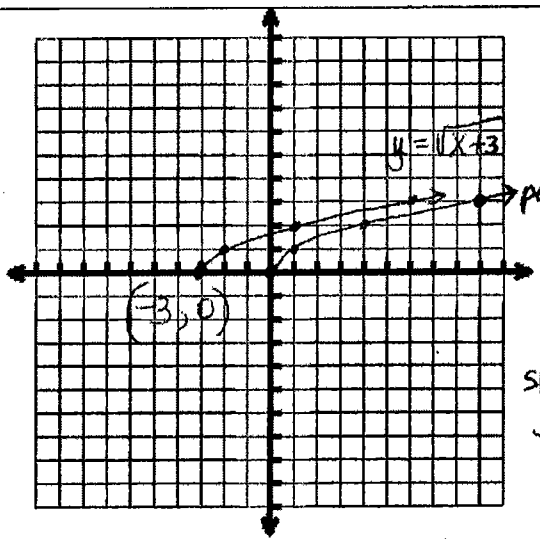
37. $\dfrac{\sqrt[3]{2xy}}{\sqrt[3]{4x^2}}=\sqrt[3]{\dfrac{y}{2x}}\cdot\sqrt[3]{\dfrac{4x^2}{4x^2}}=\dfrac{\sqrt[3]{4x^2y}}{2x}$

denom: $\sqrt[3]{8x^3}$

38. For each function, find the following and then graph. (Without a calculator, which means you should graph the parent function first!)

a. $y=\sqrt{x+3}$ (left 3)

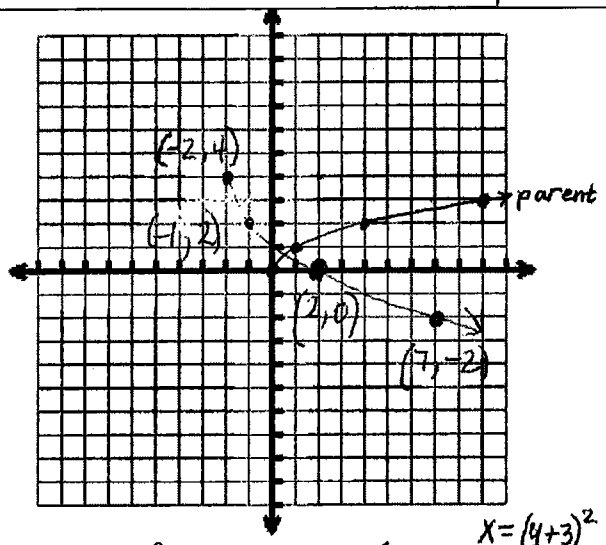
Domain: $x\geq -3$ Range: $y\geq 0$



shift all points 3 to left

b. $y=-2\sqrt{x+2}+4$ opens down/vt stretch 2, left 2, up 4

Domain: $x\geq -2$ Range: $y\leq 4$

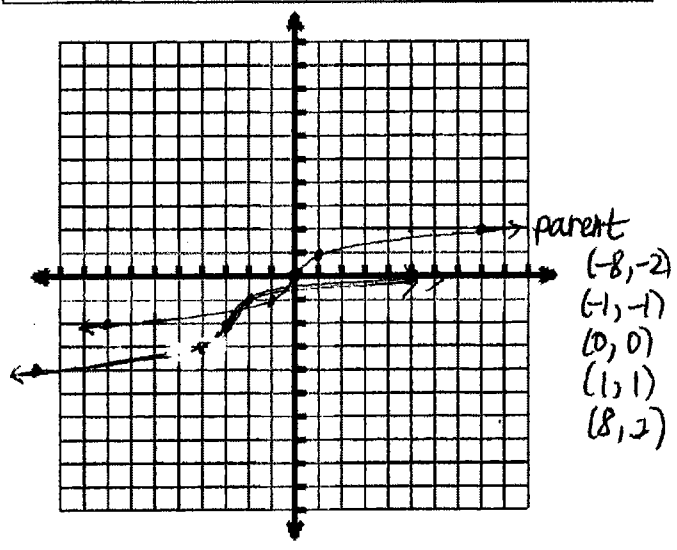


c. $f(x)=\sqrt[3]{x+3}-2$ left 3, down 2

Domain: $\mathbb{R}$ Range: $\mathbb{R}$

x-intercept(s): (5,0)

y-intercept(s): $\sqrt[3]{0+3}-2=\sqrt[3]{3}-2$



d. $g(x)=(x+3)^2$ and its inverse $g^{-1}(x)$

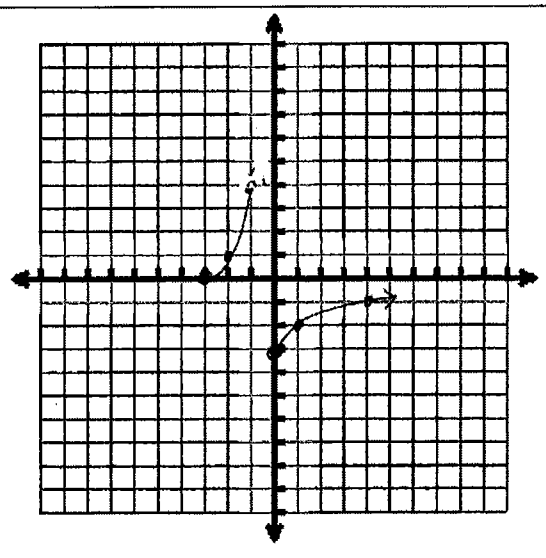
$x=(y+3)^2$

$\sqrt{x}-3=y$

$g^{-1}(x)=\sqrt{x}-3$

Domain: $x\geq -3$ Range: $y\geq 0$

Domain of $g^{-1}(x)$: $x\geq 0$ Range of $g^{-1}(x)$: $y\geq -3$



Chapter 9 – Rational Functions **Factor first!**

Multiply or Divide.

39. $\frac{x^2-5x+6}{x^2-4} \cdot \frac{x^2+3x+2}{x^2-2x-3}$

$\frac{(x-2)(x-3)}{(x+2)(x+2)} \cdot \frac{(x+2)(x+1)}{(x-3)(x+1)} = \frac{1}{1}$

$\boxed{1}$

$X \neq 2, -2, 3, -1$

40. $\frac{y^2-5y+6}{y^3} \div \frac{y^2+3y-10}{4y^2}$ ← multiply & flip the 2nd fraction

$\frac{(y-2)(y-3)}{y^3} \cdot \frac{4y^2}{(y+5)(y-2)}$

$\boxed{\frac{4(y-3)}{y(y+5)}}$

$Y \neq 0, -5, 2$

Add or Subtract. 1) create LCD 2) add numerators & write as a single fraction.

41. $\frac{1}{x^2+5x+4} + \frac{x}{3x+3}$

$\left(\frac{3}{3}\right)\frac{1}{(x+4)(x+1)} + \frac{x}{3(x+1)}\left(\frac{x+4}{x+4}\right)$

$\frac{3+x^2+4x}{3(x+4)(x+1)}$

$\frac{x^2+4x+3}{3(x+4)(x+1)} = \frac{(x+3)(x+1)}{3(x+4)(x+1)} = \boxed{\frac{x+3}{3(x+4)}}$

42. $\frac{5y}{y^2-7y} - \frac{4}{2y-14}$

$\left(\frac{2}{2}\right)\frac{5y}{y(y-7)} - \frac{4}{2(y-7)}\left(\frac{y}{y}\right)$

$\frac{10y-4y}{2y(y-7)}$

$\frac{6y}{2y(y-7)}$

$\boxed{\frac{3}{y-7}}$

Simplify.

43. $\frac{x-6}{x^2-7x+6}$

$\frac{(x-6)}{(x-6)(x-1)}$

$\boxed{\frac{1}{x-1}}$

44. $\frac{x^2-4}{x(x+2)}$

$\frac{(x+2)(x-2)}{x(x+2)}$

$\boxed{\frac{x-2}{x}}$

Solve each equation. if $\frac{a}{b}=\frac{c}{d}$, then cross multiply $ad=bc$; multiply both sides by LCD.

45. $\frac{2}{3x-5}=\frac{4}{x-5}$

$2X-10=12X-20$

$10=10X$

$\boxed{X=1}$

46. $\left[\frac{5}{2x}-\frac{2}{3}=\frac{1}{x}+\frac{5}{6}\right]6X$

$\frac{5(6X)}{2X}-\frac{2(6X)}{3}=\frac{1(6X)}{X}+\frac{5(6X)}{6}$

$15-4X=6+5X$

$9=9X$

$\boxed{1=X}$

47. $\left[\frac{3x-2}{9}-\frac{1}{6}=\frac{1}{3}\right]18$

$\frac{18(3X-2)}{9}-\frac{18(1)}{6}=\frac{18(1)}{3}$

$2(3X-2)-3=6$

$6X-4-3=6$

$6X-7=6$

$6X=13$

$\boxed{X=\frac{13}{6}}$

48. $\frac{7x+3}{x^2-8x+15}+\frac{3x}{x-5}=\frac{1}{x-3}$

$\left[\frac{7X+3}{(X-3)(X-5)}+\frac{3X}{X-5}=\frac{1}{X-3}\right](X-3)(X-5)$

$7X+3+3X(X-3)=X-5$

$7X+3+3X^2-9X=X-5$

$3X^2-3X+8=0$

use quad formula

$X=\frac{3\pm\sqrt{9-4(3)(8)}}{2(3)}$

$\boxed{\text{no real sol}}$

Simplify

49. $\frac{5}{x^2-9}+\frac{2}{x+3}$

$\frac{5}{(X-3)(X+3)}+\frac{2}{X+3}\left(\frac{X-3}{X-3}\right)$

$5+2X-6$

$\boxed{\frac{2X-1}{(X+3)(X-3)}}$

50. $\frac{1}{x^2+4x4}-\frac{2}{x^2+2x}$

$\left(\frac{X}{X}\right)\frac{1}{(X+2)^2}-\frac{2}{X(X+2)}\left(\frac{X+2}{X+2}\right)$

$\boxed{\frac{X-2X-4}{X(X+2)^2}}$

careful: subtraction

Determine where points of discontinuity are (Holes and Vertical Asymptotes) if there are any. Then find the Horizontal Asymptote.

51. $f(x)=\frac{2x-2}{x^2-1}=\frac{2(X-1)}{(X-1)(X+1)}=\frac{2}{X+1}$

HOLE $(1,\frac{2}{1+1})$ $\rightarrow(1,1)$

VA $X=-1$

52. $g(x)=\frac{(x-1)(x-2)(x-3)}{(x+2)(x-3)(x+1)}=\frac{(X-1)(X-2)}{(X+2)(X+1)}$

HOLE $(3,\frac{(2)(1)}{(5)(4)})$

Hole $(3,\frac{1}{10})$

VA $X=-2$

VA $X=-1$

Find the horizontal and vertical asymptotes and points of discontinuity, then graph.

$\frac{(2x+3)(x-5)}{(x-5)(x-1)} = \frac{2x+3}{x-1}$

53. $y = \frac{2x+3}{x-5}$

HA: $y = 2$

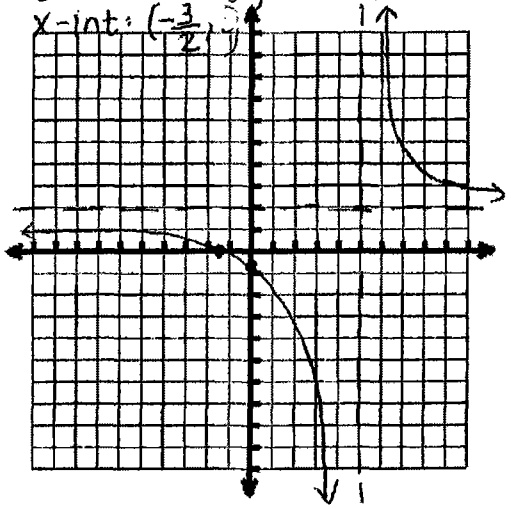
VA: $x = 5$

Holes: none

y-int: $(0, -\frac{3}{5})$

x-int: $(-\frac{3}{2}, 0)$

VA $x=5$



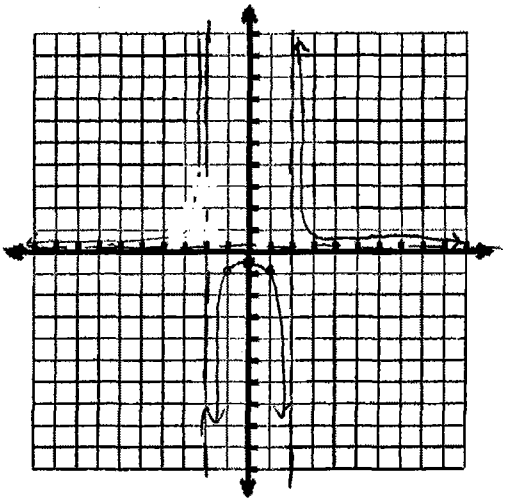
54. $f(x) = \frac{2}{x^2-4} = \frac{2}{(x-2)(x+2)}$

HA: $y = 0$

VA: $x = 2, x = -2$

Holes: none

y-int $(0, -\frac{1}{2})$



55. $y = \frac{2x^2-7x-15}{x^2-6x+5}$

HA: $y = 2$

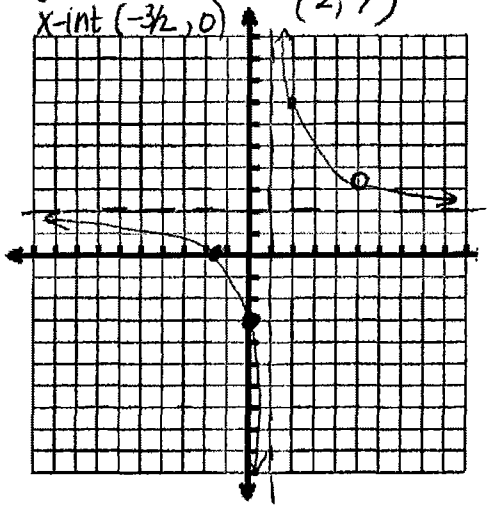
VA: $x = 1$

Holes: $(5, \frac{13}{4})$

y-int $(0, -3)$

x-int $(-\frac{3}{2}, 0)$

additional pt $(2, 7)$



56. If Jamie can paint a house in 6 hours & Kimberlina can paint a house in 9 hours, how long will it take for them to paint it together?

$\frac{6}{x} + \frac{9}{x} = 1$

	r	t	d
J	$\frac{1}{6}$	6	1
K	$\frac{1}{9}$	9	1
T	$\frac{1}{x}$	x	1

$\left(\frac{3}{3}\right)\frac{1}{6} + \frac{1}{9}\left(\frac{2}{2}\right) = \frac{1}{x}$

$\frac{5}{18} = \frac{1}{x}$

$5x = 18$

$x = 3.6$ hrs

57. A boat, which moves at 36 mi/h in still water, travels 28 mi downstream in the same amount of time that it takes to travel 20 mi upstream. Find the speed of the current.

C = speed of current

Distance = rate x time

$\begin{cases} 28 = 36t + ct \\ 20 = 36t - ct \end{cases}$

$28 = t(36+c) \rightarrow t = \frac{28}{36+c}$

$20 = t(36-c) \rightarrow t = \frac{20}{36-c}$

$\frac{28}{36+c} = \frac{20}{36-c}$

$1008 - 28c = 720 + 20c$

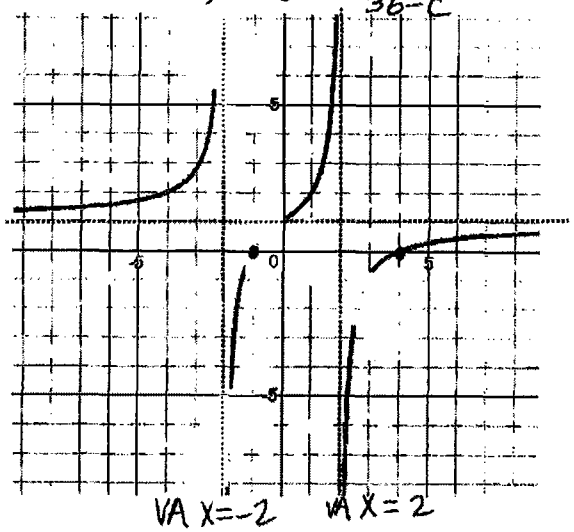
$288 = 48c$

$6 = c$

6 mi/hr

58. Find the equation of the graph.

x-int $(-1, 0), (4, 0)$



$f(x) = \frac{(x+1)(x-4)}{(x-2)(x+2)}$